

Find the general solution of the differential equation $x^2 y'' + 5xy' + 3y = \sin 2x$. $\leftarrow q = x^{-2} \sin 2x$ SCORE: ___ / 10 PTS

① POINT
ALL ITEMS
UNLESS
OTHERWISE
NOTED

$$\textcircled{\frac{1}{2}} r^2 + 4r + 3 = 0$$

$$r = -1, -3$$

$$\textcircled{\frac{1}{2}} y_h = c_1 x^{-1} + c_2 x^{-3}$$

\uparrow \uparrow
 y_1 y_2

$$W = \begin{vmatrix} x^{-1} & x^{-3} \\ -x^{-2} & -3x^{-4} \end{vmatrix} = -3x^{-5} + x^{-5} = -2x^{-5}$$

$$y_p = \left[-x^{-1} \int \frac{(x^{-2} \sin 2x) x^{-3}}{-2x^{-5}} dx \right] + \left[x^{-3} \int \frac{(x^{-2} \sin 2x) x^{-1}}{-2x^{-5}} dx \right]$$

$$= -x^{-1} \int -\frac{1}{2} \sin 2x dx + x^{-3} \int -\frac{1}{2} x^2 \sin 2x dx$$

$$= -x^{-1} \left(\frac{1}{4} \cos 2x \right) + x^{-3} \left(\frac{1}{4} x^2 \cos 2x - \frac{1}{4} x \sin 2x - \frac{1}{8} \cos 2x \right)$$

$$= -\frac{1}{4} x^{-2} \sin 2x - \frac{1}{8} x^{-3} \cos 2x$$

$$y = -\frac{1}{4} x^{-2} \sin 2x - \frac{1}{8} x^{-3} \cos 2x + c_1 x^{-1} + c_2 x^{-3}$$

$$\begin{array}{r} -\frac{1}{2} x^2 \quad \sin 2x \\ -x \quad -\frac{1}{2} \cos 2x \\ -1 \quad -\frac{1}{4} \sin 2x \\ 0 \quad \frac{1}{8} \cos 2x \end{array}$$

Find the general solution of the differential equation $y'' + 6y' + 9y = xe^{-3x} - 45\cos 6x$.

SCORE: ____ / 20 PTS

$$\textcircled{1} r^2 + 6r + 9 = 0$$

$$r = -3, -3$$

$$\textcircled{1} y_h = c_1 e^{-3x} + c_2 x e^{-3x}$$

$$y_p = x^2(Ax + B)e^{-3x} + C\cos 6x + D\sin 6x$$

$$= (Ax^3 + Bx^2)e^{-3x} \textcircled{2} + C\cos 6x + D\sin 6x \textcircled{1}$$

$$y_p' = (-3Ax^3 - 3Bx^2)e^{-3x} + 6D\cos 6x - 6C\sin 6x + (3Ax^2 + 2Bx)e^{-3x}$$

$$= (-3Ax^3 + (3A - 3B)x^2 + 2Bx)e^{-3x} + 6D\cos 6x - 6C\sin 6x \textcircled{1}$$

$$y_p'' = (9Ax^3 + (-9A + 9B)x^2 - 6Bx)e^{-3x} - 36C\cos 6x - 36D\sin 6x$$

$$+ (-9Ax^2 + (6A - 6B)x + 2B)e^{-3x}$$

$$= (9Ax^3 + (-18A + 9B)x^2 + (6A - 12B)x + 2B)e^{-3x}$$

$$\textcircled{3} -36C\cos 6x - 36D\sin 6x \textcircled{1}$$

$$+ 6y_p' = (-18Ax^3 + (18A - 18B)x^2 + 12Bx)e^{-3x} + 36D\cos 6x - 36C\sin 6x$$

$$+ 9y_p = (9Ax^3 + 9Bx^2)e^{-3x} + 9C\cos 6x + 9D\sin 6x$$

$$\textcircled{2} (6Ax + 2B)e^{-3x}$$

$$\textcircled{2} \begin{cases} + (-27C + 36D)\cos 6x \\ + (-36C - 27D)\sin 6x \end{cases}$$

$$6A = 1 \quad 2B = 0$$

$$A = \frac{1}{6} \quad B = 0$$

$$-27C + 36D = -45$$

$$-36C - 27D = 0$$

$$\rightarrow -3C + 4D = -5 \quad \times 3$$

$$-4C - 3D = 0 \quad \times 4$$

$$-9C + 12D = -15$$

$$-16C - 12D = 0$$

$$-25C = -15$$

$$C = \frac{3}{5}$$

$$D = -\frac{4}{3}C = -\frac{4}{5}$$

$$y = \frac{1}{6}x^3 e^{-3x} + \frac{3}{5}\cos 6x - \frac{4}{5}\sin 6x$$

$$+ c_1 e^{-3x} + c_2 x e^{-3x}$$

\textcircled{1}